Test of gravitational redshift with optical lattice clocks and their applications

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“Space-time information platform with a cloud of optical lattice clocks”
Optical lattice potential confines millions of neutral atoms in separate micro-traps

- Strong confinement suppresses atomic thermal motion and allows Doppler-free spectroscopy
- Tuning the lattice laser to the magic frequency $\nu_m$ realizes light-shift-free confinement at the lowest order

$$h\nu = h\nu_0 - \frac{1}{2}\Delta\alpha(\nu_m)E^2 + O(E^4), \quad \Delta\alpha(\nu_m) = 0$$

- Probing a large number of atoms improves the stability of the clock by $\sigma \propto 1/\sqrt{N}$

“Accurate” and “stable” clocks are valuable not only as a standard, but also as a quantum sensor that can probe static and dynamic phenomena
Application of optical lattice clocks to relativistic geodesy

Einstein’s theory of general relativity

“A clock in a lower place ticks slower than one in a higher place due to gravity”

- Height difference $\Delta h = 1 \text{ cm}$ causes time dilation of $\frac{\delta \nu}{\nu} \approx 10^{-18}$
- Accurate clocks become a precise probe of gravitational potential
- The clock becomes a system of elevation that defines the equipotential surface: “Quantum benchmark”

Ref. “Chronometric levelling”

Demonstration of relativistic geodesy

- Frequency comparison of remote optical lattice clocks in RIKEN and The Univ. of Tokyo

$\frac{\delta \nu}{\nu} = \frac{g \Delta h}{c^2}$

Altitude difference $\Delta h \sim 15 \text{ m}$

T. Takano et al., Nat. Photon. 10, 1038 (2016)
Remote frequency comparison between RIKEN and UT ('16)

- Measure the difference in altitude between RIKEN (Wako) and the UT (Hongo) by clock comparison
- Consistent with the results of elevation difference measured by spirit leveling
- Difference between spirit leveling and chronometric leveling by clock comparison
  - Spirit leveling: ~2 km/day, Cumulative error: \(2.5\sqrt{S/\text{km}}\text{ mm} \) (\(S\): distance)
  - Chronometric leveling: real-time measurement, no cumulative error
- Chronometric leveling can observe dynamic changes in altitude (e.g., tidal effect, crustal deformation, etc.)
- Transportability of the clock is an issue for application as a measurement tool

\[ \frac{\nu_{\text{Sr2}} - \nu_{\text{Sr1}}}{\nu_0} : 1(1) \times 10^{-18} \]
\[ \frac{\Delta \phi}{c^2} \]
\[ \nu_{\text{UT}} - \nu_{\text{Sr1}} : -1,653(7) \times 10^{-18} \]

T. Takano et al., Nat. Photon. 10, 1038 (2016)

Spirit leveling by Geospatial Information Authority of Japan (GSI)
18 frequency-stabilized lasers are required to operate two clocks
The system works only inside the laboratory
For practical applications outside laboratory, development of transportable systems is required
Transportable Optical Lattice Clocks

Spectroscopy chamber with magnetic shield (60 cm)³

Laser welded robust & compact external cavity diode lasers

Vacuum pump

1D decelerator

Main chamber

Atom oven

19’ rack mountable laser system
1D optical lattice inside a ring cavity

**Clock sequence**
1. Load atoms into lattice
2. Transport atoms into BBR shield
3. Excite clock transition
4. Transport atoms back to outside
5. Measure excitation rate
6. Feedback to laser frequency to keep resonant to Sr transition

Repeat $t_c = 1.6$ s

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**1st stage cooling**
- Allowed: 461 nm
- 2 mK

**2nd stage cooling**
- Spin forbidden: 689 nm

**Lattice trap**
- Magic WL: 813 nm
- 1 µK
- $500 \mu$m

**Moving lattice**
- Magic WL: 813 nm
- BBR shield

Transport atoms into BBR shield by moving lattice
A pair of transportable OLCs connected by a fiber link

- Laser systems with control electronics mounted on 19 inch racks
- Laser box #1: cooling (461, 496 nm), pumping (679 nm) lasers
- Laser box #2: narrow-line cooling (689 nm), lattice (813 nm), clock (698 nm) lasers
- Both clocks are connected by a noise canceled telecom fiber to send cavity-stabilized reference lasers at subharmonics from a laser distributor

Laser control electronics developed by Shimadzu Co., Ltd.
Preliminary experiment in laboratory:
*Measure height difference of 1 m by comparing two clocks*

- Lift up one of two clocks and compare their clock frequencies
- Measure gravitational time dilation for 1 m height difference with an averaging time of a few minutes
Preliminary experiment in laboratory:

Measure height difference of 1 m by comparing two clocks

Time dilation for 1 m height difference due to general relativity is resolved by comparing optical lattice clocks with a few minutes of averaging time.
Test of gravitational redshift

- Optical lattice clocks set at 0 m and 450 m in TOKYO SKYTREE
- Connect two clocks with an optical fiber and measure gravitational redshift for height difference of 450 m
  - Gravitational redshift: $\delta \nu_{\text{redshift}} \sim 21$ Hz ($\delta \nu/\nu_0 \sim 5 \times 10^{-14}$)
  - Uncertainty of redshift: $10^{-18}/(5 \times 10^{-14}) \sim 10^{-5}$
- Measure height difference $\Delta h$ by laser ranging and GNSS
  - Evaluate parameter $\alpha$:
    
    $$\frac{\delta \nu_{\text{redshift}}}{\nu_0} = (1 + \alpha) \frac{g \Delta h}{c^2}$$

- Previous test of gravitational redshift
  - Pound-Rebka experiment (Harvard tower: $\Delta h = 23$ m)
    $$|\alpha| < 0(10^{-2})$$ (PRL 4, 337 (1960))
  - Gravity Probe A mission, NASA (Launched H-maser: $\Delta h = 10,000$ km)
    $$|\alpha| \sim 1.4 \times 10^{-4}$$ (PRL 45, 2081 (1980))
  - Galileo satellites, ESA (Atomic clocks on elliptic orbits: $\Delta h \approx 8,500$ km)
    $$\alpha = (0.19 \pm 2.48) \times 10^{-5}$$ (PRL 121, 231101 (2018))
    $$\alpha = (-0.9 \pm 1.4) \times 10^{-5}$$ (PRL 121, 231102 (2018))

- Test of GR with ground-based ($\Delta h \approx 0.5 \text{ km} \ll 10^4$ km) experiment using optical lattice clocks
Installation of two transportable OLCs in TOKYO SKYTREE (Oct. 2, 2018)

Period of experiment: Oct. 3, 2018 - Apr. 9, 2019
A pair of optical lattice clocks at 0 m and 450 m floor

- Optical fiber link (700 m) with fiber noise cancelation system to compare clocks at 0 m and 450 m
- Automated operation with remote access
- Realization of stable operation even with diurnal environmental temperature change of $> 10 \, ^\circ\text{C}$ at TOKYO SKYTREE
Test of GR: gravitational redshift & height measurement by surveying

\[ \frac{\delta \nu}{\nu} = (1 + \alpha) \frac{g \Delta h}{c^2} \]

Clock comparison

Two ways of surveying

① GNSS (Geospatial authority of Japan)

② Laser ranging
Laser ranging were performed by laser distance meter using an aperture near the central pillar of the tower

※ Short distance:
Spirit leveling by GSI
Test of GR: gravitational redshift & height measurement by surveying

Test of GR: gravitational redshift & height measurement by surveying

\[ \frac{\Delta \nu}{\nu_1} (10^{-18}) \]

<table>
<thead>
<tr>
<th>Measurement period</th>
<th>Height difference ( \Delta h ) (m)</th>
<th>Clock ( f ) difference</th>
<th>( \Delta \nu/\nu_1 ) (10(^{-18}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>MJD 58,414 – 58,417</td>
<td>GNSS 452.649(39) Laser ranging 452.632(13)</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>MJD 58,420 – 58,422</td>
<td>452.652(39) Laser ranging 452.631(13)</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>MJD 58,571 – 58,577</td>
<td>– Laser ranging 452.596(13)</td>
<td>49,337.8(4.3)</td>
<td>–</td>
</tr>
<tr>
<td>MJD 58,599 – 58,623</td>
<td>– –</td>
<td>–</td>
<td>-0.3(4.7)</td>
</tr>
</tbody>
</table>

\[ \alpha = \frac{\Delta \nu}{\nu_1} \frac{\Delta h}{\tilde{g} \Delta h} - 1 = 1.4(9.1) \times 10^{-5} \]

Transportable optical lattice clocks on vehicle

(Toyota wagon, modified by AISIN Co., Ltd.)

- Optical lattice clock (19-inch rack x2 + physics package) loaded on a wagon
- Clock can be operated by providing power supply
- By connecting an optical fiber to transfer the clock frequency, the clock can be compared with reference master clocks
View of the interior of the vehicle with clock setup

Rack #1
- Laser sources for spectroscopy (clock 698 nm, lattice 813 nm, cooling 689 nm)
- Reference cavity

Rack #2
- UHV chamber for spectroscopy
- Peltier controller for BBR shield
- Sequencer, PC, etc.

Rack #3
- Laser sources for atom cooling (cooling 461 nm, 496 nm, 679 nm)
- Clock transfer laser and cavity (clock $(2\lambda_c) : 1.4 \mu m$)
Clock operation and spectroscopy in vehicle

![Clock operation and spectroscopy in vehicle](image)

**Mathematical Expressions**

- Clock transition
- Ramsey spectrum with $T = 400$ ms

**Approximations**

- $\Delta \nu \approx 1.3$ Hz

**Graphs**

- Number of atoms (a.u.) vs. Frequency (kHz)
- Excitation probability vs. Frequency (Hz)
Frequency comparison between on-vehicle clock and lab. clock

\[ \Delta \nu = \nu_2 - \nu_1 \]

- 1.4 \(\mu\)m stable laser
- Laborotary clock \(\nu_1\)
- On-vehicle clock \(\nu_2\)
- Ground (Outside)
- \(\Delta h \approx 5\) m
- \(\Delta h \approx 20\) m

Compare 2 clocks in the lab.

\[ \nu_{\text{vehicle}} - \nu_{\text{lab}} = 228.1 \text{ mHz} \]

Faster elevation difference measurements than with GNSS surveying

\[ \sigma_y(\tau) \approx 8 \times 10^{-17} / \sqrt{\tau} \]

\[ \rightarrow \sigma_{\Delta h}(\tau) \approx 80 \text{ cm} / \sqrt{\tau} \]

\[ \text{Vehicle-Lab} \]

\[ \text{Lab-Lab} \]

\[ \text{GNSS} \]

1cm in 1.5h

1cm in 24h

Developed by AISIN
Transportable clocks with long-distance fiber link

• Long distance fiber link from Tokyo to Tohoku area (in preparation by NTT, telecom company)
  – Fiber link from Wako (RIKEN) / Hongo (UT) / Atsugi (NTT) to Mizusawa-Esashi (VLBI observatory) with PLC (planar lightwave circuits) repeaters
  – Distance: 500 km, fiber length: 800 km

• Transport on-vehicle clock and compare with reference clock (in RIKEN/UT) using a long-distance fiber link

• Measure gravitational potential change for geodetic applications
  – ex. slow uplift of the ground after the earthquake in Tohoku area (postseismic relaxation, a few cm/yr)

Ref. Y. Tanaka and H. Katori, “Exploring potential applications of optical lattice clocks in a plate subduction zone,” J. Geodesy 95, 93 (2021)


**Summary**

- Test of gravitational redshift in Tokyo Skytree
  - Develop a pair of transportable OLCs and demonstrate stable operation outside laboratory
  - Comparing clocks at 0 m and 450 m with an uncertainty of $10^{-18}$ tested the gravitational redshift with an uncertainty of $10^{-5}$

- Development of an optical lattice clock on vehicle
  - Frequency comparison with a laboratory clock measured the height difference with cm-precision in a few hours of averaging
  - Future applications of on-vehicle clock to relativistic geodesy using a long-distance fiber link